

VOLTAGE  
DIVIDERS

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# DC AND LOW FREQUENCY AC RATIO MEASUREMENTS

BY

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**esi**



# DC AND LOW FREQUENCY AC RATIO MEASUREMENTS

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The resistive voltage, current and impedance ratio measuring devices discussed include simple tapped resistance strings, decade dividers, attenuators, bridges and ratio sets.

Transformer devices include simple fixed voltage and current transformers, decade ratio transformers, and transformer type bridges.

Accuracy, stability and traceability of calibration are considered.

Various types of dc and ac detectors are reviewed, and characteristics of sensitivity and impedance matching, response speed and damping, noise limitations, and spurious signal rejection are considered.

A discussion of techniques and pitfalls in ratio measurement includes grounding, shielding, dc and ac guarding, and the effects of ambient temperature, power dissipation, and voltage and current levels.

## VOLTAGE RATIO

Any voltage ratio measurement implies the existence of two voltages to be compared. We could connect voltmeters to each voltage and divide one reading by the other. But, since we want only the ratio between them, the measurement of the actual voltages will only cause extra work. More important, it will introduce additional sources of error in the ratio measurement. This is true even if we use a high accuracy digital voltmeter. Therefore, when we want to compare two voltages, it is better to use a calibrated adjustable divider.

Typical circuits for such comparisons are shown in Figure 1. Each circuit contains an adjustable voltage divider, the tap setting being represented by the letter S.

For the circuits on the left a null detector is observed while the voltage divider is adjusted until its tap is the same voltage as the circuit to which it is connected.

If both voltage sources have a very high impedance, no current can be drawn from either one. In this case, an auxiliary voltage source,  $e_0$ , can be used to supply power to the divider as shown at the right in the figure. This voltage is adjusted so that no current is drawn from either voltage source when the ratio is being measured.

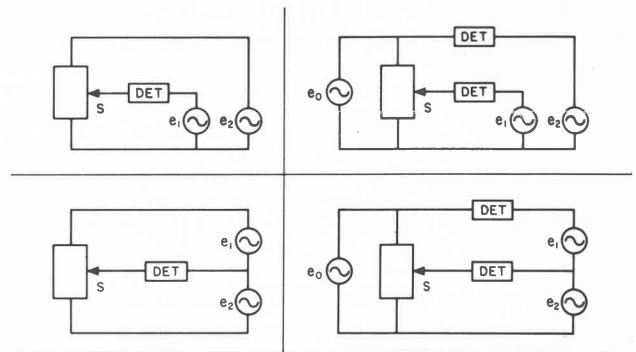


FIGURE 1. VOLTAGE RATIOS

The voltage ratio to be measured is usually produced by a divider, a transformer, an operational amplifier or something of the sort.

## DIVIDER RATIO

If we replace the two voltages in Figure 1 with another divider, we obtain the circuit of Figure 2. When the detector indicates a null, the voltages on both divider taps are equal, so the ratios are equal.

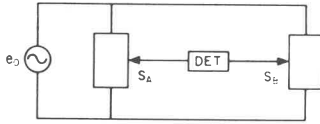


FIGURE 2. DIVIDER COMPARISON

## BRIDGE RATIO

Each divider may consist of two impedances connected in series. This is the circuit of the impedance bridge of Figure 3.

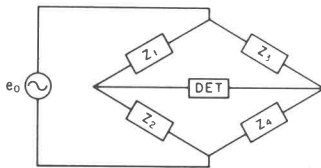


FIGURE 3. IMPEDANCE BRIDGE

## DIVIDER AND BRIDGE SETTING

If we concentrate on one half of either of these circuits, as in Figure 4, we see the essential difference between the pair of dividers and the bridge.

|         | RESISTOR | TRANSFORMER |
|---------|----------|-------------|
| DIVIDER |          |             |
| BRIDGE  |          |             |

FIGURE 4. DIVIDER AND BRIDGE RATIOS

The divider setting  $S$  indicates the ratio of a part of the voltage to the total. The bridge is calibrated to indicate the ratio between the two parts into which the total voltage is divided, which is the ratio of the impedances. Both resistor and transformer circuits can be calibrated to indicate either type of ratio.

It is also possible to make capacitor voltage dividers. These are particularly useful because they are very accurate at higher frequencies. Since we are discussing dc and low frequency ratio work, we shall concentrate on resistive and transformer type dividers with the understanding that at these frequencies, capacitor dividers can be used in a manner quite similar to resistor dividers. For example a capacitance bridge usually consists of two capacitors which make one divider and two resistors or a transformer which make the other.

## FIXED TAP DIVIDERS

In practical instruments voltage dividers take many forms. The simplest is a string of two or more resistors connected in series with terminals at their junctions. In Figure 5 we see such a circuit.

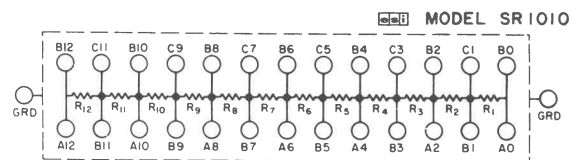


FIGURE 5. TAPPED DIVIDER

Here two terminals are provided at each junction so that we can use four-terminal measuring techniques to eliminate the effects of wire lead resistances. Figure 6 shows a photograph of this unit.

This type of divider is extremely useful for high accuracy standardization of resistors and determination of

resistance ratios. Its use in calibrating resistive voltage dividers will be discussed later.

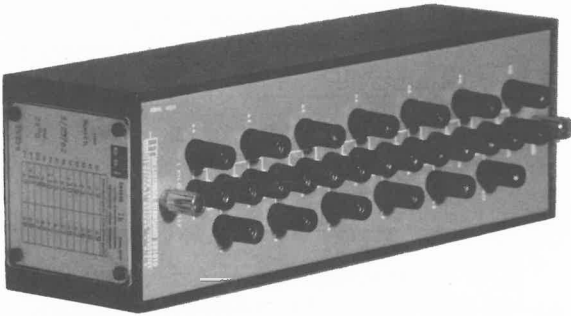


FIGURE 6. RESISTANCE TRANSFER STANDARD

Figure 7 shows another type of voltage divider -- a voltage calibrator, which is used to compare any of several

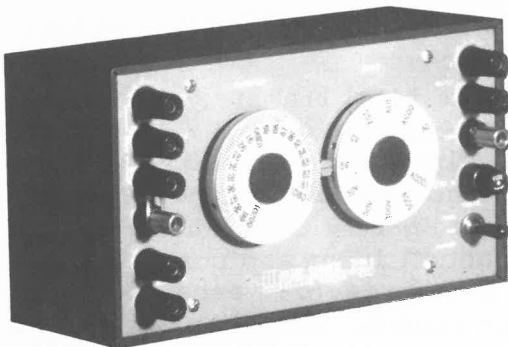


FIGURE 7. VOLTAGE CALIBRATOR  
nominal voltage values with the voltage of a standard cell. The circuit of this instrument is shown in Figure 8.

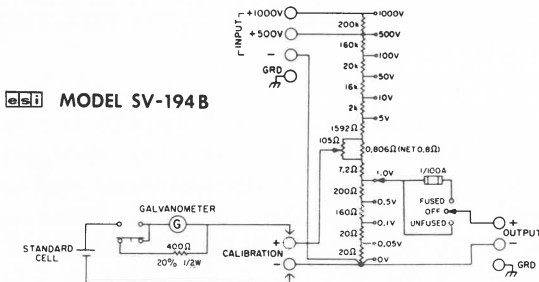


FIGURE 8. CALIBRATOR CIRCUIT

When the adjustable divider is set to the certified value of the standard cell and the input voltage is adjusted so that the detector in series with the standard cell shows a null, the output voltage is very accurately equal to the nominal value selected.

## DECADE DIVIDERS

By far the most popular type of voltage divider for high accuracy ratio measurement is the decade divider, either in its resistive or transformer form. Several typical decade dividers are shown in Figure 9.

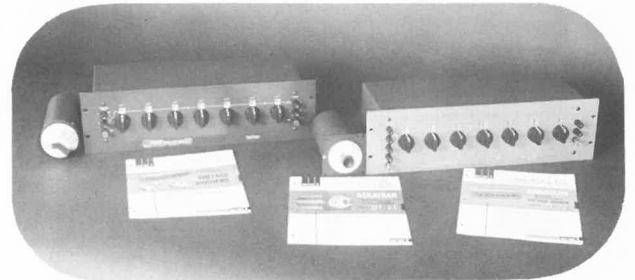


FIGURE 9. VOLTAGE DIVIDERS

With these dividers a voltage can be split into a fraction which is indicated in decimal form. The best dividers at the present state of the art have seven or more adjustable decades. Figure 10 shows a set of calibration curves for a very high accuracy resistive decade divider.

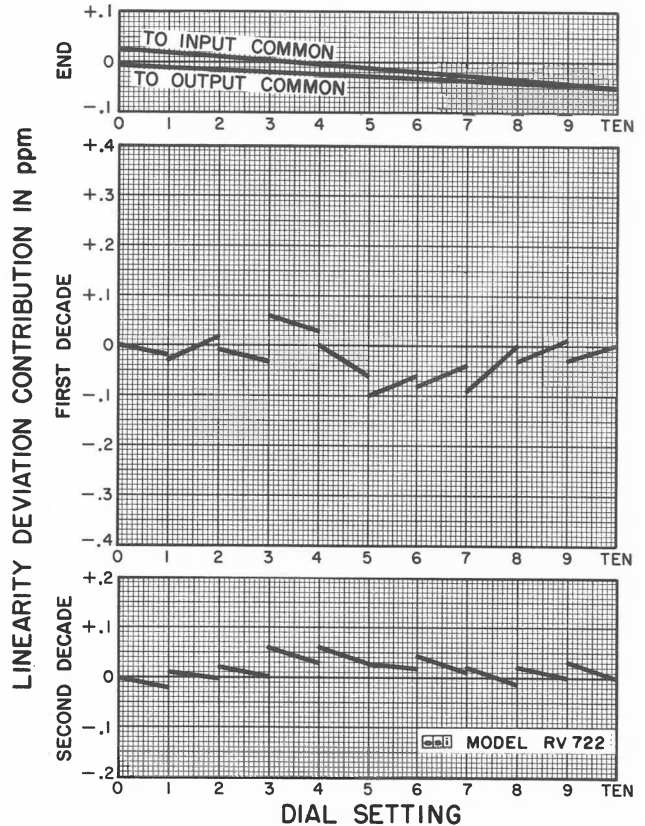


FIGURE 10. CALIBRATION OF A RESISTIVE DIVIDER

The best transformer decade dividers produce similar accuracies (see Figure 11) for low-frequency ac measurements.

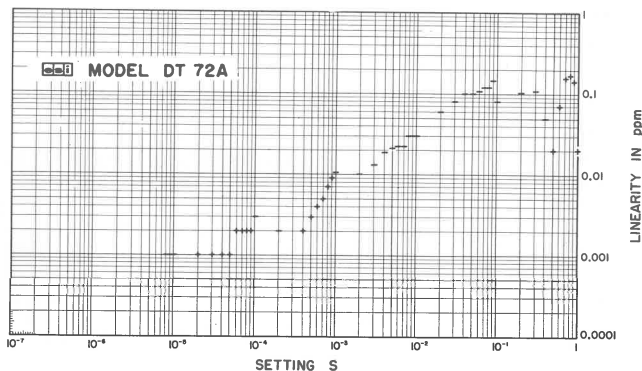


FIGURE 11. CALIBRATION OF A TRANSFORMER DIVIDER

## TRANSFORMER DECADE DIVIDERS

A decade transformer can be represented by the equivalent circuit of Figure 12.

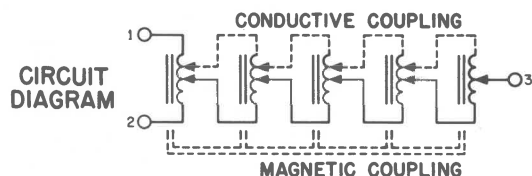


FIGURE 12. DECADE TRANSFORMER

Here each decade is represented as a transformer winding which is normally divided into ten parts. Sometimes eleven or twelve decade steps are provided to allow extra positions at either end. Each knob controls the setting of one of these decades.

Adjacent decades may be coupled by either of two methods. The decades may be independent transformers with conductive coupling from two taps to both ends of the following winding, or there may be windings for two or more decades on one transformer core.

When a common core provides magnetic coupling between decades, only one switch contact per decade is necessary. Extra windings or extra switch contacts and conductive coupling paths may be used to improve the accuracy in practical designs.

## EQUIVALENT CIRCUIT

The equivalent circuit of any decade transformer can be represented as shown in Figure 13.

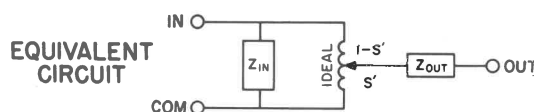


FIGURE 13. RATIO TRANSFORMER

The ideal transformer in the equivalent circuit is a device which has infinite open circuit impedance, zero short circuit impedance and is used to specify the transformer ratio. The actual input and output impedances can be represented separately as shown in the figure. The transformer ratio,  $S'$ , is very nearly equal to the turns ratio. The quantity  $S'$  is actually a vector quantity which can be calibrated. It has a magnitude which may be slightly different from the nominal value (the turns ratio) and there may be a slight phase shift between the input and output voltages.

The calibration certificate for a decade transformer gives the deviation of the ratio  $S'$  from its nominal value in both magnitude and phase. The certified ratio will be obtained if no load current is drawn from the output. If load current is drawn we must determine whether its effect can be neglected.

## INPUT IMPEDANCE

The input impedance curves of a typical high accuracy decade transformer standard are shown in Figure 14.

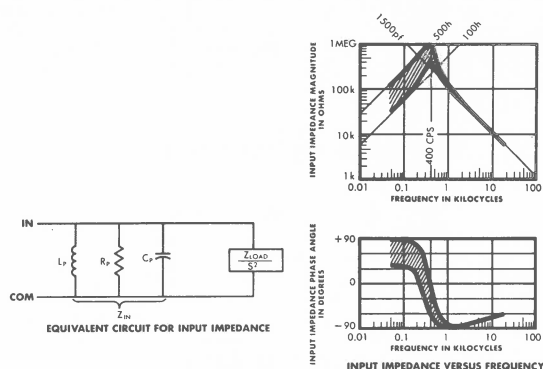


FIGURE 14. INPUT IMPEDANCE

The input impedance varies with frequency. It is inductive at lower frequencies, passes through resonance, and becomes capacitive at higher frequencies. The transformer ratio  $S'$  remains quite accurate over this whole range of frequencies. The effect of this input impedance resonance is to minimize the current which the transformer draws from the voltage source; the accuracy is not affected.

## OUTPUT IMPEDANCE

The output impedance curves for the same transformer are shown in Figure 15.

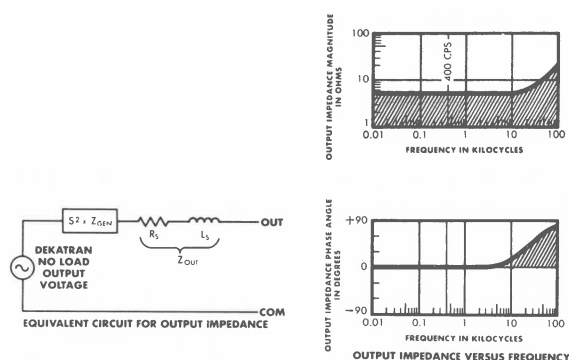


FIGURE 15. OUTPUT IMPEDANCE

Note here that at low frequencies the output impedance is simply the dc resistance of the winding. At higher frequencies, the leakage reactance causes some inductance to be added to the output impedance.

## DECADE TRANSFORMER APPLICATIONS

There are many practical applications for decade transformer voltage dividers. The ratios which they produce can be used in various voltage and impedance comparison circuits.

## DIVIDER CALIBRATION

A decade transformer is frequently used to calibrate another decade transformer or for ac calibration of a resistive decade divider. These divider calibration circuits are shown in Figures 16 and 17.

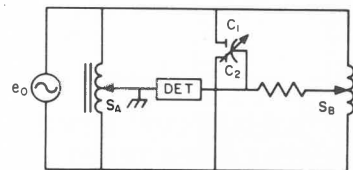


FIGURE 16. TRANSFORMER DIVIDER CALIBRATION

In these circuits, phase compensating capacitors are used to adjust for any phase difference between the two dividers. If the divider on the right has low reactive output impedance, such as a transformer, resistance is added in series with its tap as shown in Figure 16, so that its output resistance will be much larger than its reactance. A capacitor is connected from the detector end of the resistor to either end of the divider, or a differential capacitor is connected to both ends of the divider as shown. This capacitance can be used to draw a small amount of current through the resistor. This causes a small quadrature voltage drop across the resistor. If this voltage is extremely small compared to the total voltage across the dividers, it can be considered to be quite accurately in quadrature and will have a negligible effect on the magnitude calibration. If a larger difference in phase is to be corrected, careful attention must be paid to see that the current through the

resistance is exactly at right angles to the applied voltage. Any phase difference from  $90^\circ$  in the current through the resistor will cause a voltage drop in phase with the transformer voltage being calibrated, and will affect the magnitude ratio measurement.

Usually very sensitive detectors must be operated with one of their two terminals grounded to prevent stray pickup at their input terminals. The tap of the transformer at the left is grounded so that this end of the detector can be grounded.

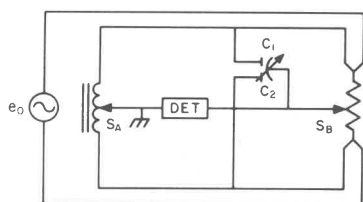


FIGURE 17. RESISTIVE DIVIDER CALIBRATION

The calibration of a resistive divider is very similar except that the divider output is already resistive in nature, therefore the capacitors can be directly connected across either or both halves of the divider as shown in Figure 17. In this case the added capacitance combines with the internal capacitance across the two halves of the divider to make a capacitance divider. At null this capacitance divider has the same ratio as the resistance divider. This technique minimizes errors resulting from internal and connection capacitances in the resistive divider being tested.

## TRANSFORMER RATIO

Figure 18 shows how to calibrate a transformer voltage ratio accurately using a decade standard divider. The calibration circuit for any step-down ratio is essentially the same as that for calibration of another decade divider. To calibrate a step-up ratio without loading effects, it is necessary to

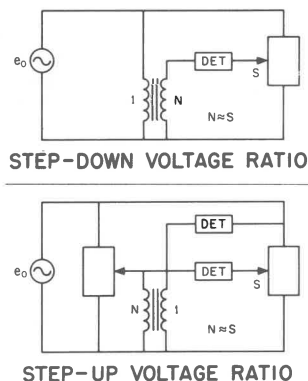


FIGURE 18. TRANSFORMER RATIO MEASUREMENT

provide an auxiliary supply such as another divider to drive the lower voltage winding of the transformer being tested. This voltage must be adjusted so that the voltage on the higher voltage winding is exactly the same as that applied to the standard divider. The standard divider is then adjusted to measure the relative voltage applied to the low voltage windings. The auxiliary divider does not need to be calibrated, but it does have to have sufficient resolution to adjust the output voltage accurately. Some phase shift adjustment is likely to be required on both the standard and the auxiliary divider. Techniques similar to those in Figures 16 and 17 should be used; they are omitted from Figure 18 for simplicity.

## IMPEDANCE RATIO

A decade transformer can be used with a capacitance standard to make a

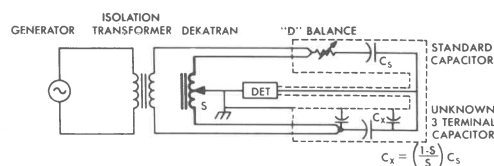


FIGURE 19. CAPACITANCE BRIDGE

capacitance bridge of extreme accuracy over a very wide capacitance range as shown in Figure 19. Similar circuits

can be used for other impedance comparisons.

## SYNCHRO AND RESOLVER TESTS

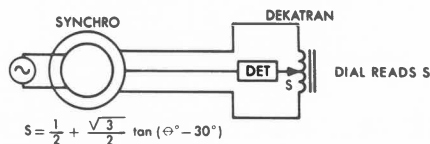


FIGURE 20. SYNCHRO TEST

Synchro and resolver components and systems can be checked to accuracies of one second or better. Figure 20 shows one of the synchro test circuits.

## CURRENT RATIO

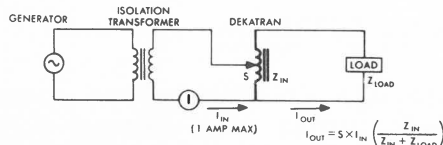


FIGURE 21. CURRENT DIVISION

The reversed transformer divider makes an accurate and useful current divider, as shown in Figure 21.

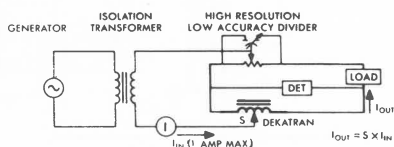


FIGURE 22. ACCURATE CURRENT DIVISION WITH LOAD

The circuit of Figure 22 can be used to make accurate current division even with high load impedances.

## RESISTIVE DECADE DIVIDERS

Decade transformers are probably the most accurate reference dividers available for the middle audio frequencies. However, to achieve the ultimate in their accuracy the frequency range is somewhat limited. For lower frequencies and for dc a resistive divider

is required. The resistive decade divider circuit usually used as a laboratory standard for high precision work is shown in Figure 23. This is known as the Kelvin-Varley circuit.

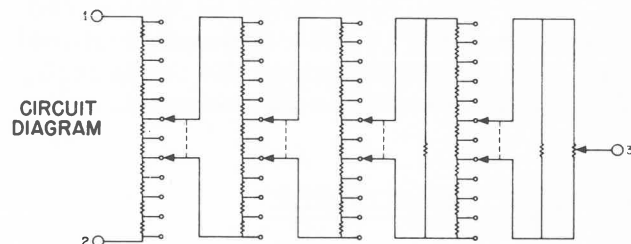


FIGURE 23. KELVIN-VARLEY CIRCUIT

In this circuit, each decade contains eleven resistors instead of ten. The resistance to the right of each decade is made equal to the value of two resistors in the decade. When this resistance to the right is connected across any two adjacent resistors of the decade, the resistance across the combination will equal that of one resistor of the decade. The end-to-end resistance of the decade will be that of ten equal resistors, rather than eleven.

If it is more convenient to use an interpolating decade whose total resistance is higher than two resistors of the preceding decade, a shunt resistor can be connected across the interpolating decade to reduce the combined value to that of the two shunted resistors. The decades at the right end of Figure 23 use this shunting technique.

## EQUIVALENT CIRCUIT

The equivalent circuit of the Kelvin-Varley divider is shown in Figure 24. A Kelvin-Varley divider equivalent circuit



FIGURE 24. KELVIN-VARLEY DIVIDER

cuit looks exactly like a simple tapped

resistor except that it has additional resistance inserted in series with the tap.

## INPUT RESISTANCE

The input resistance is measured from the input to the common terminal with the output terminal disconnected. The input resistance is constant.

## OUTPUT RESISTANCE

The output resistance is measured from the output terminal to the common terminal with the input shorted. It varies with the setting of the divider. Equivalent circuits and curves of output resistance are shown in Figure 25. Note

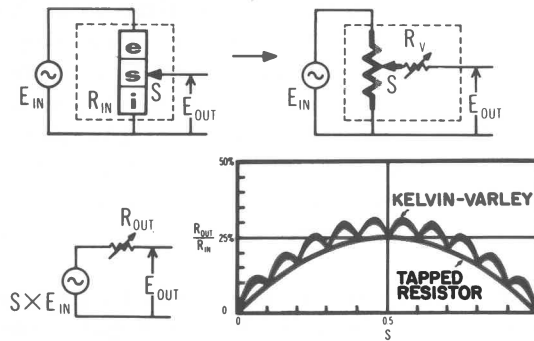


FIGURE 25. OUTPUT RESISTANCE

that the large smooth parabola is the output resistance which would be shown by a simple tapped resistor. The scallops added on top of this are the effect of the later decades. You will note that there are 10 small parabolas on top of the big one for the second decade effect. The third decade adds ten small parabolas on each of these, etc. The thickness of the line on each small parabola represents the extent of these additional variations.

## KELVIN-VARLEY APPLICATIONS

Some typical Kelvin-Varley applications are shown in Figures 26 through 30.

## VOLTAGE DIVISION

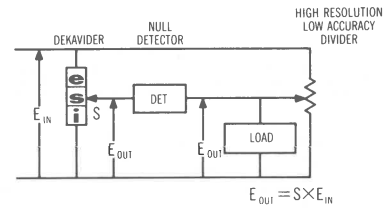


FIGURE 26. ACCURATE VOLTAGE DIVISION WITH LOAD

A voltage divider can be used to provide a calibrated, adjustable voltage reference from an accurate fixed input voltage. If a burden current must be supplied without disturbing the voltage calibration an auxiliary divider can be used as shown in Figure 26.

## CURRENT DIVISION

Although we have talked about the resistive divider as a voltage divider it is also applicable as a current divider as shown in Figure 27.

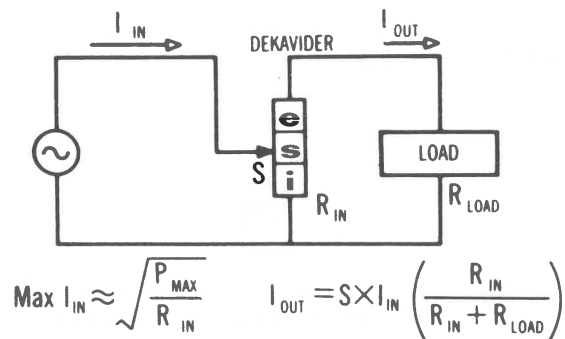


FIGURE 27. RESISTIVE CURRENT DIVIDER

As a current divider, the constant input resistance of the Kelvin-Varley divider gives it an advantage over a decade transformer for small currents into non-zero impedance loads. If the load resistance and the input current are kept constant, the load current will be accurately proportional to the divider setting, no matter how high the load resistance. The load current at full scale must, of course, either be

measured or calculated, unless the load resistance is essentially zero.

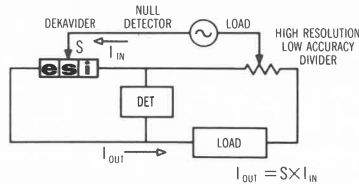


FIGURE 28. ACCURATE CURRENT DIVISION WITH LOAD

If the load resistance is not known, or is not constant, an auxiliary divider can be used in the circuit of Figure 28 to make the divider indicate directly the ratio of load current to input current.

### IMPEDANCE RATIO

A calibrated divider ratio can be used for measuring impedance values by comparison with a known standard as shown in Figure 29.

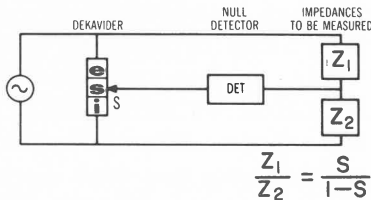


FIGURE 29. IMPEDANCE RATIO MEASUREMENT

### SYNCHRO AND RESOLVER TESTS

Synchro and resolver bridges are calibrated by divider comparisons. A

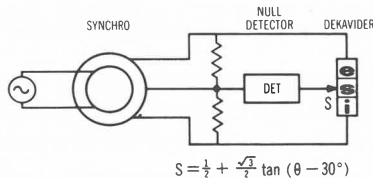


FIGURE 30. SYNCHRO TEST

synchro bridge can be made of a resistive divider and a pair of load balancing resistors.

### THINGS TO WATCH OUT FOR

To get the best accuracy out of any very precise divider we must take various precautions. The first thing to consider is the effect of stray series and shunt impedances.

### END RESISTANCE

Figure 31 shows how we can compare two resistive voltage dividers without errors being introduced by the resistances of the leads and terminals at the ends of the dividers. The detector is connected between the two divid-

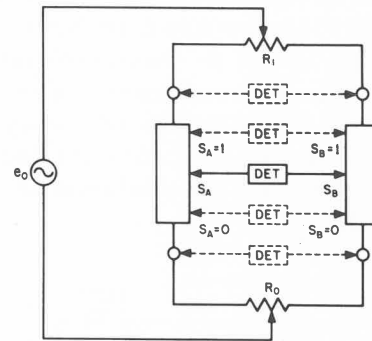


FIGURE 31. END RESISTANCE CORRECTION

ers at their respective tap settings,  $S_A$  and  $S_B$ . The detector will usually not read zero when both dividers are set to either zero or full scale because of different lead and contact resistances in the two dividers. We can purposely introduce resistances joining the ends of the two dividers and supply the current through taps on these auxiliary tapped resistors. We can adjust these taps to set the voltages at the ends of the dividers equal.

Note that we have a choice. We can adjust the potentiometers so that the terminals of the dividers are set at the same voltage, or we can adjust the dividers so that their outputs at their maximum and minimum settings are exactly equal. Which adjustment we choose will depend upon the application

we intend to make of our divider. Normally for greatest precision the dividers should be calibrated relative to the highest and lowest settings. To do this we should adjust  $R_0$  with the detector connected to the taps of the dividers and all their dials set to minimum, and  $R_1$ , with all dials at maximum. An independent measurement of end resistance can be made to find the corrections to be applied when the voltage output is to be referred to the terminals.

### SHUNT IMPEDANCE

Shunt impedances, both resistive and capacitive, always exist in any device. In dividers of the accuracy we are considering we need to be somewhat careful about dc leakage paths. For ac dividers even more care should be taken with the effects of stray capacitance. An equivalent circuit for the principal leakage impedances is shown in Figure 32.

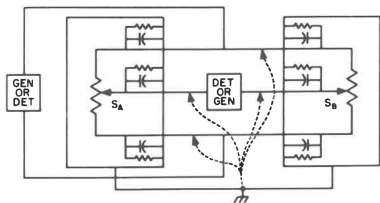


FIGURE 32. LEAKAGE IMPEDANCES

### GROUND CONNECTION

When comparing dividers we have four choices of where in the divider comparison circuit we connect ground. By "ground" we mean the cases of the dividers. Whichever ground point we select, the two adjacent parts of the dividers will be shunted by leakage resistance and capacitance. If one divider has a much lower input resistance than the other, the ground should be connected to the tap of the low resistance divider, so that the shunt impedances will all be placed across the two portions of the low resistance divider. If

both dividers are of equal impedance and they are being used at very low settings, the ground should be placed at the common (bottom) end. It is usually much easier to isolate a generator (a battery or an output transformer) from ground than it is to isolate the input of the sensitive detector. The generator and detector can often be interchanged to minimize leakage problems.

### EXTERNAL GUARD DRIVE

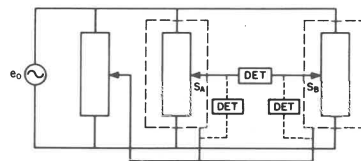


FIGURE 33. GUARDING

Figure 33 shows what to do if the selection of grounding point is not sufficient to maintain the desired accuracy. Here a third uncalibrated divider with sufficiently high resolution is connected to drive the cases of the two dividers to the potential of the taps of the dividers, as indicated by an auxiliary detector connection. When a null balance is obtained between the two divider taps there will be no voltage difference between either divider tap and its case. The effect of all six leakage impedances shown above will have been removed from the measurement circuit - - they are either across zero voltage or their currents are being supplied by the guard drive divider.

### NOT A CALIBRATED RESISTOR

In using a Kelvin-Varley divider, note that it cannot be used as a simple variable resistor. The resistance between the output and common terminals will not correspond accurately to the dial setting because of the added varying output resistance of the later decades. However, there are many circuits, including bridge circuits, in

which the variable resistance can be placed at a point where it does not enter into the calibration of the circuit.

## WHAT POWER DOES

When we use dividers we always must be careful to operate at temperature, voltage, current, power, etc.. which does not cause errors. The Kelvin-Varley circuit has a particular peculiarity as shown in Figure 34.

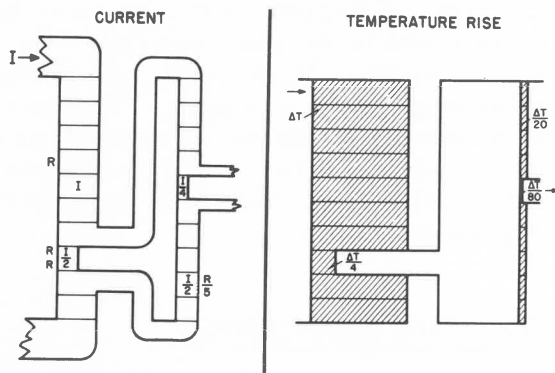


FIGURE 34. CURRENT AND TEMPERATURE DISTRIBUTION

This figure shows how the current and power divide in the resistors from one decade to the next. The temperature rise is almost directly proportional to the applied power. The uneven power distribution results from the shunting of two of the 11 resistors in one decade by the next decade. These two resistors do not get as hot as the rest in the string. Thus, in a Kelvin-Varley circuit, it is not enough to match resistor temperature coefficients. If the divider is to be run at a fairly high power, they must have very close to a zero temperature coefficient because they will be heated unequally by the input power. If we were worried simply about ambient temperature, then temperature coefficient matching would be adequate. The effects of input power on a statistical sample of typical high quality dividers is shown in Figure 35. Dividers are calibrated and often used at low power. The mean  $\pm$  one standard deviation did not exceed three parts per

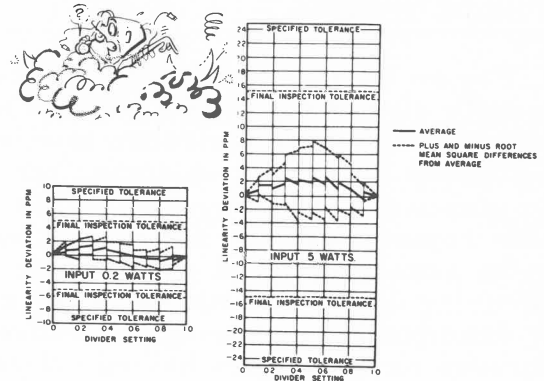


FIGURE 35. POWER SHIFT

million with an input of 0.2 watt but it increased to eight parts per million with 5 watts of input.

## BE CAREFUL

General precautions in any measurement are the same. Always observe any limitations such as voltage or current coefficient, the effects of power and ambient temperature, the effects of the humidity, pressure, electrolytic potential, leakage paths with moisture content, magnetic fields, electrostatic fields or high voltage corona. Thermoelectric potentials should be watched for, measured and corrected for. Care should be taken in precision measurements that the initial conditions of calibration, or the conditions for which the unit was designed, are duplicated as accurately as possible in the laboratory. If not, the particular effect not duplicated should be measured and corrections made if they are needed.

## DETECTORS

So far, we have shown many circuits for divider applications and divider comparisons with a simple box marked DET to represent a sensitive detector for the small unbalance signals. The selection of an appropriate detector is of utmost importance for high accuracy measurement work.

## AC DETECTORS

For ac measurements the detector is nearly always tuned and exhibits a fairly high  $Q$ . This is needed to eliminate pick-up of stray frequencies other than measurement frequency. This tuning rejects hum and harmonics which may be generated if the elements in the measuring device are not quite linear. For example in measuring iron-core inductors or capacitors having voltage coefficients, harmonic signals are often generated which do not balance at the same settings as the fundamental. Also, a narrow bandwidth detector reduces the thermal noise from the circuit to which the detector is connected.

## DC DETECTORS

For dc signal detection, two basic detector types are commonly used, galvanometers, and modulators (chopper or other) which supply ac amplifiers. The present state of the art leaves a decision between the two devices somewhat arbitrary. Both galvanometers and chopper type detectors are capable of very excellent results, but both require a good many precautions in their use.

## GALVANOMETERS

Galvanometers have been used for dc detection for many years, but they are somewhat inconvenient to use largely because of mechanical problems. A very sensitive galvanometer is extremely susceptible to vibration and other mechanical disturbances. Recently, however, a very excellent galvanometer has been immersed in oil to improve its mechanical stability and a photo electric unit used to greatly amplify its motion.

## ELECTRONIC DETECTORS

The popular trend in recent years has been toward modulator type detectors,

particularly those using mechanical or solid state choppers. The dc signal is switched on and off or reversed in polarity by the chopper to convert it to an ac square wave signal, which can then be amplified by a low noise ac amplifier. These chopper units have indeed proved to be excellent detectors, achieving detection sensitivities very near the thermal noise limit but they too have some problems. Stray ac signals can cause spurious responses and erroneous answers so care must be taken in the circuit to exclude stray ac pickup. Chopper detectors are being designed to be less sensitive to stray ac signals than those which were available a few years ago. Chopper type detectors capable of signals near theoretical noise level are available at substantially less cost than an equivalent galvanometer.

## DETECTOR MATCHING

The matching of detector sensitivity and input impedance to the requirements of a bridge or divider circuit involves more than meets the eye.

## EQUIVALENT SOURCES

Looking at Figure 36 we can see that any divider comparison or bridge

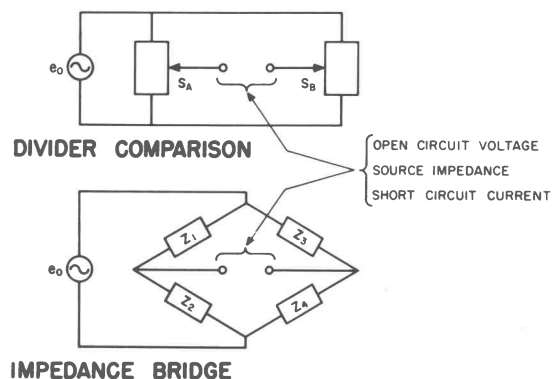


FIGURE 36. OUTPUT CIRCUIT

circuit can be looked at from its detector terminals as an open circuit voltage in series with a source impedance - - a

Thevenin equivalent circuit; or as a short circuit current in parallel with the same impedance - - its Norton equivalent circuit. In this equivalent representation the impedance is essentially constant as long as the circuit is anywhere near the null balance. The voltage or current in the equivalent circuit goes to zero at true null and rises to some level when the circuit is unbalanced a small amount. It is necessary to be able to see the voltage or current produced by the amount of unbalance corresponding to the accuracy or resolution of the dividers or bridge elements involved.

### SOURCE DESCRIBING POINT

It is particularly informative to plot any such equivalent Thevenin or Norton generator on a special kind of graph paper shown in Figure 37. Here,

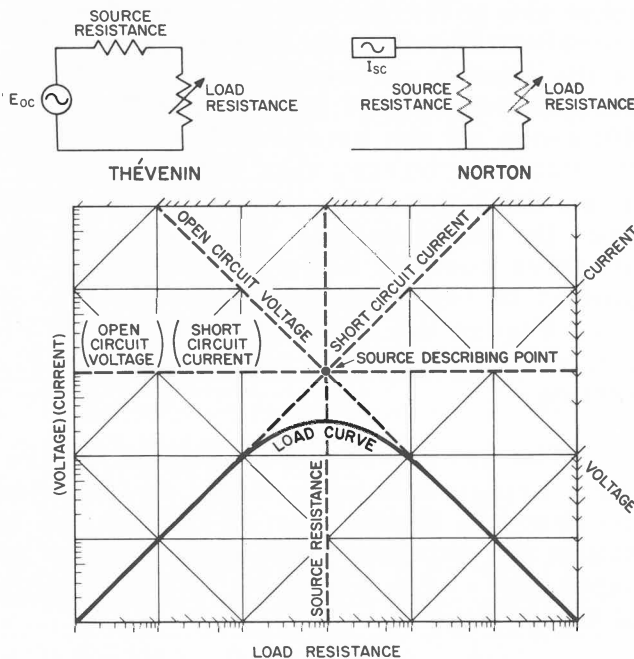


FIGURE 37. SOURCE REPRESENTATION

resistance is shown along the bottom of the graph paper, power vertically, and the voltage and current lines are the diagonals. When either type of equivalent circuit (or the real source

it represents) is connected to any load resistance, the amount of power can be plotted versus load resistance to produce the curve shown in the figure. This load curve always has exactly the same shape - - it can be drawn by means of a template. The curve shows the well-known impedance matching laws - - the maximum power into the load is obtained when the load resistance equals the source resistance, and this maximum power will be one fourth the product of the open circuit voltage and the short circuit current.

The open circuit voltage, the short circuit current, and the source resistance all intersect at a single point. This can be called the "source describing point". This single point on the graph paper serves to completely specify the source in a manner which later can be used to find the amount of voltage or current actually supplied to any load.

### DETECTOR GRAPHS

Now let us turn our attention to the detector. It is also possible, as shown

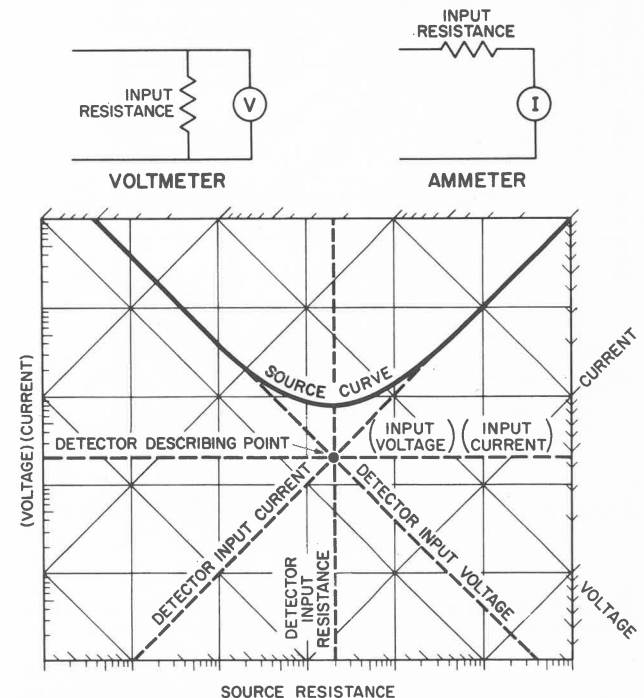


FIGURE 38. DETECTOR REPRESENTATION

in Figure 38, to describe any specified amount of deflection of a detector by a point. The detector describing point represents its input resistance, voltage and current for this deflection. We can also draw a curve which represents the locus of source describing points which would produce the specified detector indication. The same template can be used to draw both the source and detector curves.

Different amounts of deflection are represented by different curves, as illustrated in Figure 39. If we plot the describing point for a source on such a set of curves for any detector we can immediately read from the curves the amount of meter deflection which will be produced.

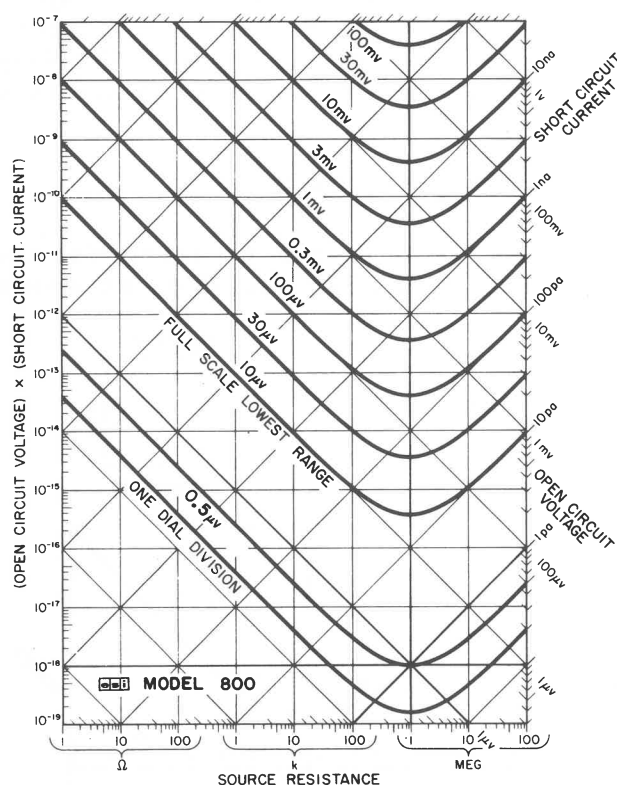


FIGURE 39. DETECTOR CURVES

## PERFORMANCE GRAPHS

At any given setting of a bridge or other null-type circuit a single describing point represents the output signal

for a specified unbalance. The output describing points for all settings can therefore be presented in a graph as shown in Figure 40.

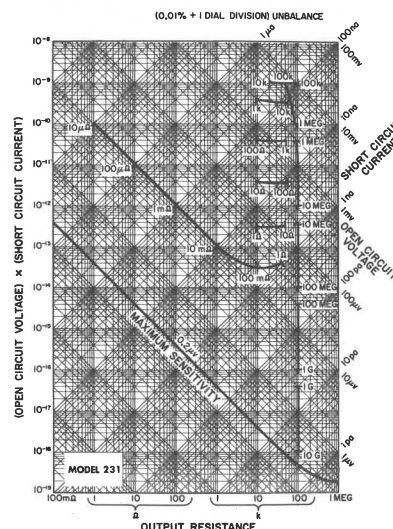


FIGURE 40. BRIDGE PERFORMANCE

Such a plot can be used to determine the performance of the bridge in combination with any detector. A plot of the maximum sensitivity curve for a sample detector is included in Figure 40; since all the bridge curves lie above the detector curve, this detector will allow the bridge to achieve full accuracy over its whole range. If a whole set of detector curves, as in Figure 39, is plotted on translucent paper and laid over Figure 40, the actual deflection can be read from the curves for any setting.

The curves shown in Figure 40 are for a bridge unbalance equal to its rated accuracy; a similar set of curves can be drawn for any other specified amount of unbalance -- for example, the resolution of the bridge.

## DETECTOR COMPARISON

Such detector curves can be used to compare one detector with another as shown in Figure 41. Here a rather typical collection of both ac and dc detector maximum sensitivity curves are

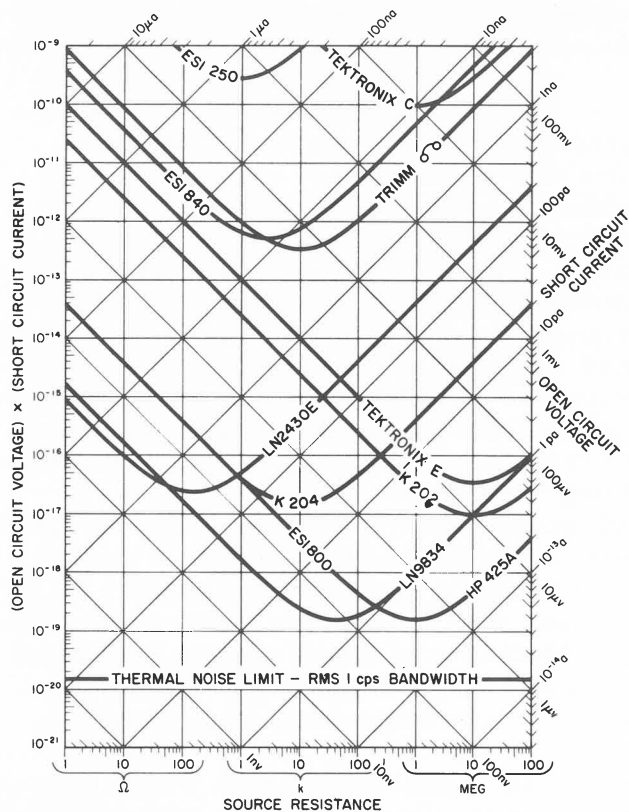


FIGURE 41. DETECTOR COMPARISON

shown. It can be seen that some detectors are more sensitive for low impedance sources, others for high impedance sources. Any bridge describing point graph can be laid over such a collection of detector curves to see which detectors have enough sensitivity for the bridge settings and accuracies of interest.

## GALVANOMETER

If a low-resistance source is connected to a galvanometer it will be extremely overdamped or sluggish. If the source resistance is too high, the galvanometer will be underdamped, and the needle will swing back and forth a long time before settling down to a final indication. To avoid these problems the source resistance can be kept constant by adding series or shunt resistance. If this is done the detector curves are modified as shown in Figure

42. The dotted line at the bottom shows the signal which would produce a minimum visible deflection of one fifth dial division if the extra series resistance were removed.

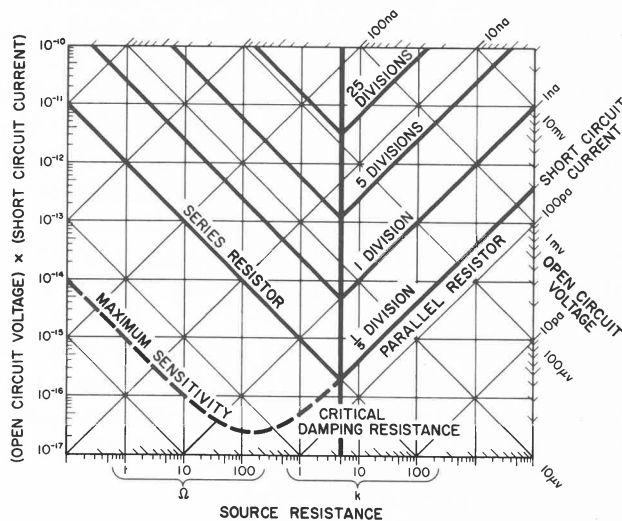


FIGURE 42. GALVANOMETER WITH CRITICAL DAMPING

Along the dotted portion of the curve, however, the galvanometer would be so highly overdamped that it would take a very long time, perhaps many seconds, for the meter to reach the indicated deflection.

## DETECTOR SENSITIVITY TESTS

A test circuit which will determine the actual input signal versus source impedance necessary to cause a discernable indication is shown in Figure 43. The ac test circuit shows an ad-

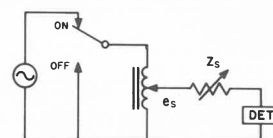


FIGURE 43. AC TEST CIRCUIT

justable voltage source with an adjustable impedance, usually a resistance, connected in series. To obtain the detector threshold curve, set  $Z_s$  at each

particular value and find the smallest value of  $e_s$  for which the detector will show a change when the generator is switched on and off. It is best to have one operator observe the detector and another one throw the switch on and off at random, to avoid prejudice and to make a statistical sampling of what percentage of the time the observer can correctly call the operation of the switch. For each impedance  $Z_s$  one point on the curve will be obtained (the intersection of  $Z_d$  and  $e_s$ ). If the detector noise varies with source impedance this curve will not have the "standard template" shape shown in the preceding figures.

The dc test circuit shown in Figure 44 is the same except that here we may add a time delay, a resistor and capacitor, so that when the switch is thrown

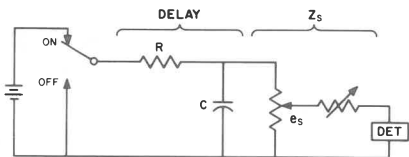


FIGURE 44. DC TEST CIRCUIT

the voltage will rise and fall slowly, perhaps with a time constant of one second or longer. This will be useful if we expect to use the detector with a source that may have transient signals which can obscure the signal for a corresponding period. This time delay also aids in finding out whether one can detect slowly changing input signals.

#### DIVIDER CALIBRATION AND TRACEABILITY

Anyone in a well equipped laboratory can calibrate a voltage divider. He can use comparison techniques to determine the "alike-ness" of nominally equal impedance elements, then connect them in combinations to give predictable voltage ratios. The resistance transfer standard shown in Figures 5 and 6 is ideally suited for this application. An

extension of this technique is applicable to ac divider calibration.

We hear a lot these days about traceability of units to the National Bureau of Standards. In case of a divider, however, there is no unit to trace to the National Bureau of Standards. It is not necessary to trace ratio measurements to any outside source since they are dimensionless quantities and no units of measurement are involved. What we must do is to compare one fraction of the voltage in a divider to the other fraction, and this is something that we can do ourselves with adequate equipment. On the other hand, many laboratories may find it convenient to trace their measurements to a divider which has been calibrated either by the National Bureau of Standards or by some other laboratory having adequate intercomparison facilities.

For highest precision work, however, it is often advisable to do your own ratio calibrating. The accuracy of ratio calibration which can be achieved may exceed the long term stability of any certified standard divider. With resistive dividers, a new calibration at the same working temperature is more reliable than a "certified value" found even a few hours before.

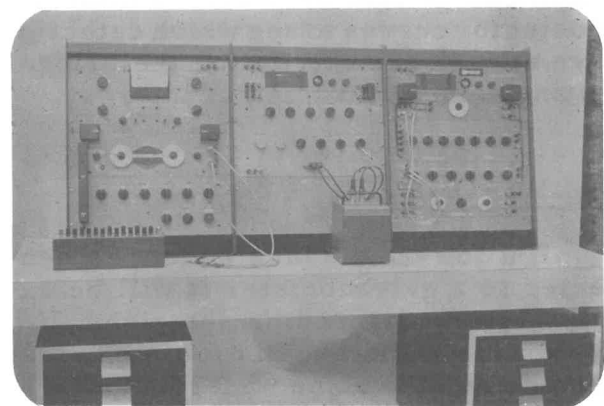


FIGURE 45. TRACEABILITY CONSOLE

Figure 45 shows a measurement console designed for making highly accurate, traceable measurements of resistance, capacitance and ratio at dc

and low frequencies. The unit on the left is a precision, guarded, four-terminal resistance measuring system, which can trace resistance measurements to certified Thomas-pattern one-ohm resistors. The center unit is a precision capacitance bridge which calibrates relative to certified standard capacitors. The right-hand panel is for ratio calibration - its standard dividers can either be certified by NBS or other outside laboratories, or directly calibrated through the use of intercomparison techniques using the bridges and standards in the units to the left.

### KELVIN-VARLEY CALIBRATION

The setting of a decade divider is the nominal ratio of the output voltage to the input voltage. The accuracy of the divider at any setting is expressed as the deviation of the actual ratio from this nominal value. This deviation can be expressed as a fraction of either the input voltage or the output voltage.

LINEARITY DEVIATION ( $\Gamma$ ) is the difference between the actual output and the nominal output expressed as a fraction of the input voltage:

$$\Gamma = \frac{E_{OUT} - SE_{IN}}{E_{IN}} = \frac{E_{OUT}}{E_{IN}} - S$$

LINEARITY is the maximum magnitude of the linearity deviation.

OUTPUT DEVIATION ( $\Delta$ ) is the difference between the actual output and the nominal output expressed as a fraction of the nominal output voltage:

$$\Delta = \frac{E_{OUT} - SE_{IN}}{SE_{IN}} = \frac{E_{OUT}}{SE_{IN}} - 1 = \frac{\Gamma}{S}$$

OUTPUT ACCURACY is the maximum magnitude of the output deviation.

A Kelvin-Varley divider can be accurately calibrated by measuring the linearity deviation of each decade relative to a ten-step reference standard divider. These measurements can then be tabulated or plotted so that the deviation at any dial setting can be calculated simply by adding the contributions from each decade.

This technique can be shown by calibrating the simple two-decade divider shown in Figure 46. For each of

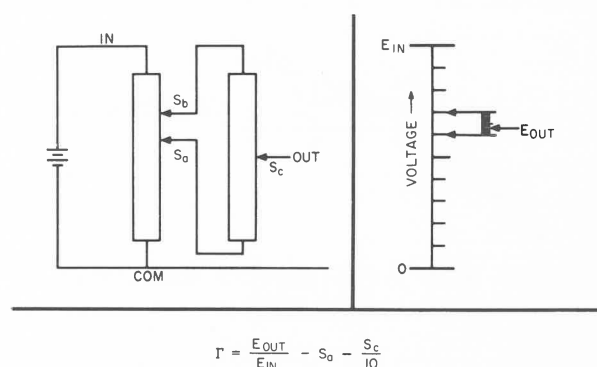


FIGURE 46. TWO DECADES

the ten settings of the first decade we shall make two measurements of output linearity deviation - one with the second decade set at zero, and one with it set at maximum. If the output divider were a perfectly linear interpolator between each of these pairs of output voltages, the linearity deviation ( $\Gamma$ ) of the whole divider could be specified graphically by plotting the measured points and joining each pair by a straight line, as shown in Figure 47.

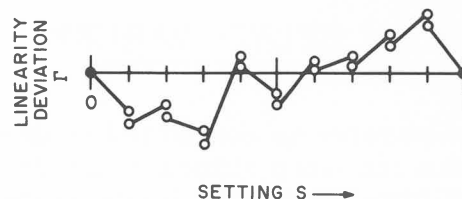


FIGURE 47. ACCURACY OF FIRST DECADE

We can make a separate calibration of the linearity deviation of the output decade relative to its own full scale range - - that is, relative to its output at zero and ten. In the two-decade divider, the range of this decade is one-tenth the range of the whole divider, therefore we can divide each of these readings by ten to express their contribution to the linearity deviation of the whole divider. These contributions are added to those from the first decade to obtain the deviation of the whole divider at any setting, as shown graphically in Figure 48.

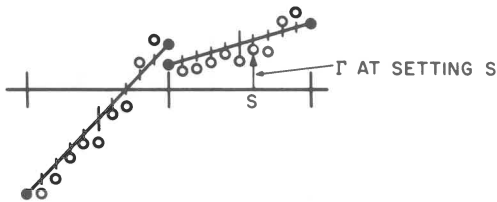


FIGURE 48. ACCURACY OF TWO DECADES

For a divider with more decades, however, it is simpler to plot separately the contribution to linearity deviation from each decade and then to add the readings at particular settings as required.

The practical calibration of Kelvin-Varley dividers is accomplished with the circuit of Figure 49. The decade

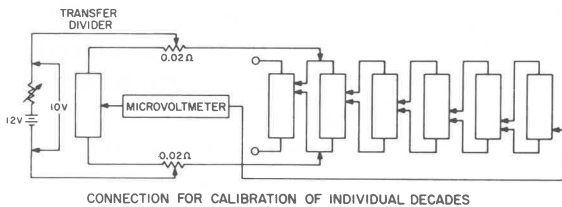


FIGURE 49. KELVIN-VARLEY CALIBRATION

to be calibrated is connected in parallel with a ten-step standard divider through low-resistance lead-compensating potentiometers. The figure shows the connection for calibrating the second decade. In this circuit, the ESI Model SR 1010 Resistance Trans-

fer Standard (Figures 5 and 6) can serve as an exceptionally accurate reference standard divider, since its tap ratios can be calculated from resistance inter-comparison measurements to a linearity of better than 2 parts in 10 million.

A high-impedance microvoltmeter is used to read the difference between the Kelvin-Varley output and the standard divider tap. The lead-compensating potentiometers shown in Figure 49 are adjusted so that the microvoltmeter indicates no voltage difference between the standard and Kelvin-Varley dividers when both are set at their maximum or their zero settings. At each setting of the decade under test, two microvoltmeter readings are taken - - one with all the decades to the right set at zero, and one with all decades to the right set at maximum. The ratio of each reading to the applied voltage represents the deviation of the Kelvin-Varley from the standard, expressed as a fraction of the full-scale range of the decade under test. These readings are then corrected for the calibration of the standard and divided by the appropriate power of ten to convert them to linearity deviation contributions expressed as a fraction of full-scale for the whole divider.

| RANGE OF DECADE | FIRST DECADE | SECOND DECADE  | THIRD DECADE     | FOURTH DECADE        | FIFTH DECADE           | SIXTH DECADE |
|-----------------|--------------|----------------|------------------|----------------------|------------------------|--------------|
| 0 TO 0X*        | 0.0 TO +0.6  | 0.00 TO -0.10  | 0.000 TO -0.050  | 0.0000 TO +0.0001,0  | 0.000,00 TO +0.000,20  | 0.000,000    |
| 1 TO 1X         | -0.3 TO 0.0  | +0.05 TO -0.14 | +0.003 TO -0.050 | -0.007,0 TO -0.006,0 | +0.000,50 TO +0.000,90 | -0.000,140   |
| 2 TO 2X         | -0.7 TO +0.4 | -0.03 TO -0.10 | +0.030 TO -0.020 | +0.003,0 TO +0.000,0 | +0.000,70 TO +0.000,90 | -0.000,011   |
| 3 TO 3X         | -0.5 TO +0.5 | -0.07 TO -0.11 | +0.060 TO +0.010 | +0.004,0 TO +0.008,0 | +0.000,60 TO +0.000,60 | -0.000,040   |
| 4 TO 4X         | +0.1 TO +1.1 | +0.06 TO +0.05 | +0.080 TO +0.030 | +0.013,0 TO +0.012,0 | +0.000,10 TO +0.000,00 | -0.000,020   |
| 5 TO 5X         | +0.5 TO +1.4 | +0.21 TO +0.14 | +0.100 TO +0.035 | +0.019,0 TO +0.019,0 | +0.000,30 TO +0.000,90 | -0.000,060   |
| 6 TO 6X         | +0.5 TO +1.0 | +0.21 TO +0.09 | +0.090 TO +0.010 | +0.021,0 TO +0.023,0 | -0.000,20 TO +0.000,40 | -0.000,090   |
| 7 TO 7X         | +0.4 TO +0.8 | +0.20 TO +0.02 | +0.080 TO 0.000  | +0.020,0 TO +0.020,0 | +0.000,70 TO +0.000,80 | -0.000,100   |
| 8 TO 8X         | 0.0 TO +0.3  | +0.14 TO +0.01 | +0.060 TO -0.010 | +0.023,0 TO +0.021,0 | +0.000,10 TO +0.000,20 | -0.000,080   |
| 9 TO 9X         | -0.4 TO 0.0  | +0.14 TO 0.00  | +0.050 TO 0.000  | +0.015,0 TO 0.000,0  | -0.000,03 TO 0.000,00  | -0.000,120   |

\* X - means that the decades to the right of the one being measured are set to maximum. For example 0.4X for the second decade indicates a dial setting of 0.049,999.

LINEARITY DEVIATION MEASUREMENTS OF AN ESI MODEL RV-622 DEKAVIDER® DECADE VOLTAGE DIVIDER

FIGURE 50. LINEARITY DEVIATION TABLE

The contributions to linearity deviation from the various decades can be tabulated as shown in Figure 50, or plotted as pairs of points joined by straight lines as in Figure 51. Note

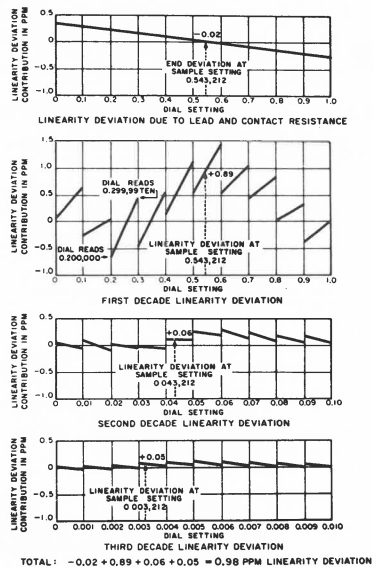


FIGURE 51. LINEARITY DEVIATION GRAPH

that the first and last reading for each decade has been made equal to zero by the adjustment of the lead-compensating potentiometers, so that the plot for the decade will represent the deviation from perfect interpolation between the highest and lowest outputs that can be obtained from that decade and the decades to the right of it. The deviation at the ends of this range is included in the calibration of the next decade to the left. Thus, an important feature of this method is that the calibration includes the effect of all lead resistances and reproducible contact resistances.

In reading such calibration graphs, the settings of all dials to the right of a given decade must be included in the

interpolation along a line segment. In Figure 51, for example, at a setting of 0.543,212, the first decade graph reads +0.89 at 0.543,212, the second decade reads +0.06 at 0.043,212, the third decade reads +0.05 at 0.003,212. The linearity deviation contributed by the last three decades is so small that it has been considered negligible and has not been plotted. Adding these readings gives an overall linearity deviation of +1.00 ppm at this setting, or an actual ratio of 0.543,213,00. This calculation calibrates the whole divider as an interpolator between its zero and full-scale output values.

If we want to calibrate the divider relative to the voltage at its input terminals, we can measure the two end voltage drops with the microvoltmeter and plot their effect as shown in the top graph in Figure 51. This shows an additional contribution to linearity deviation of -0.02 ppm at 0.543,212, making the total deviation relative to the input terminals +0.98 ppm at this setting, or an actual ratio of 0.543,212,98.

## CONCLUSION

Voltage ratio devices provide us with equipment for high accuracy voltage, current, and impedance comparisons. The present state of measurement science and commercially available materials have made possible both resistive and transformer dividers of demonstrable one ppm and better performance. Thus laboratories throughout the world can calibrate their equipment relative to high accuracy national standards by means of accurate ratio equipment and a minimum number of stable transportable standards.

MORE ABOUT VOLTAGE DIVIDERS

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